

A New Approach to LES Modeling

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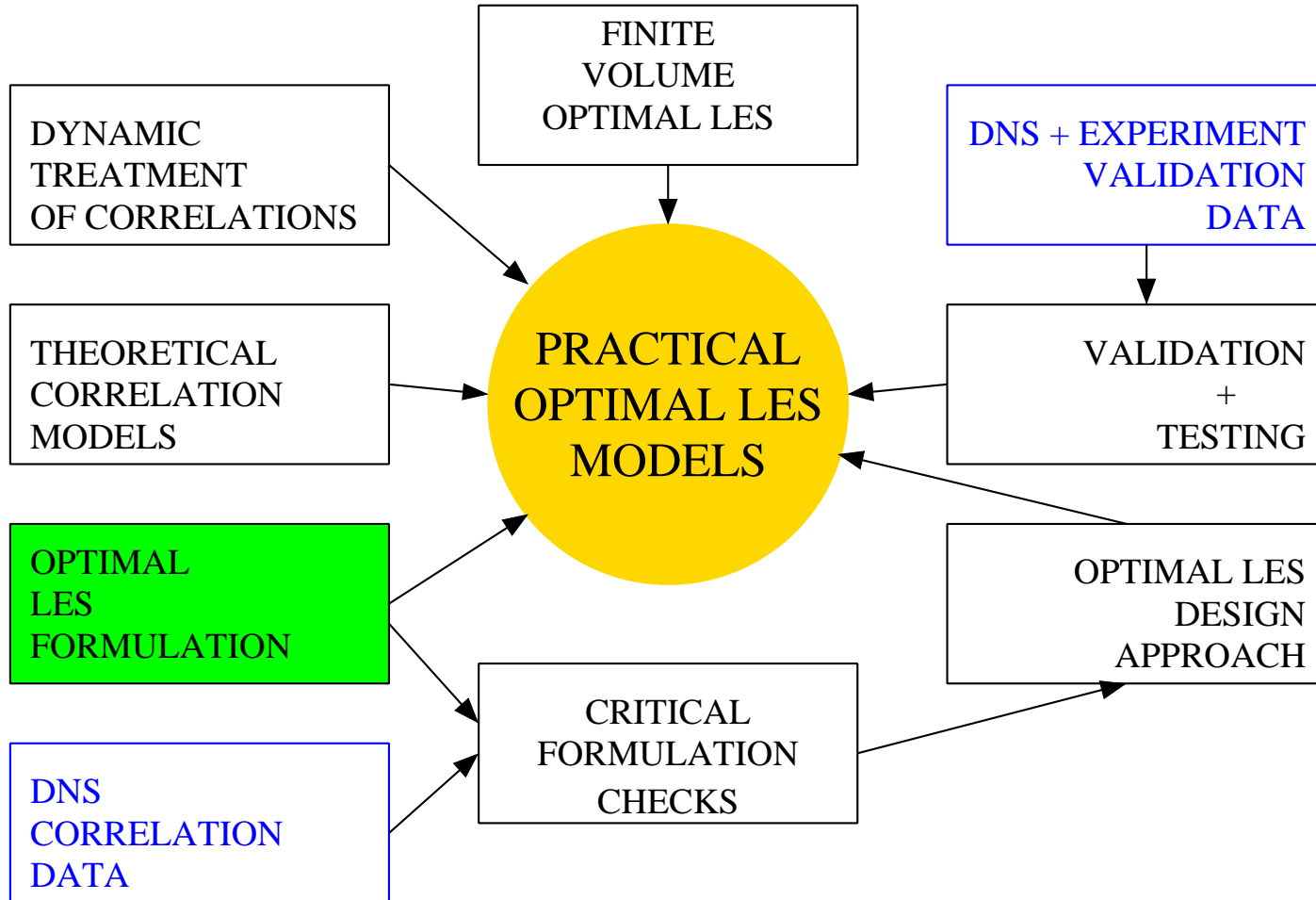
Optimal LES:

Trading in the Navier-Stokes Equations
for Custom Designed Discrete LES

Large Eddy Simulation

- Simulate only the largest scales of High-Reynolds number turbulence
 - Models of small scales required
- Numerous models developed recently
 - E.g. scale similarity, dynamic, structure function, stretched vortex, deconvolution
- Difficulties remain
 - Wall-bounded turbulence
 - Impact of numerical discretization
- LES is for making predictions!
 - Predict (some) statistical properties of turbulence
 - Predict large-scale dynamics of turbulence

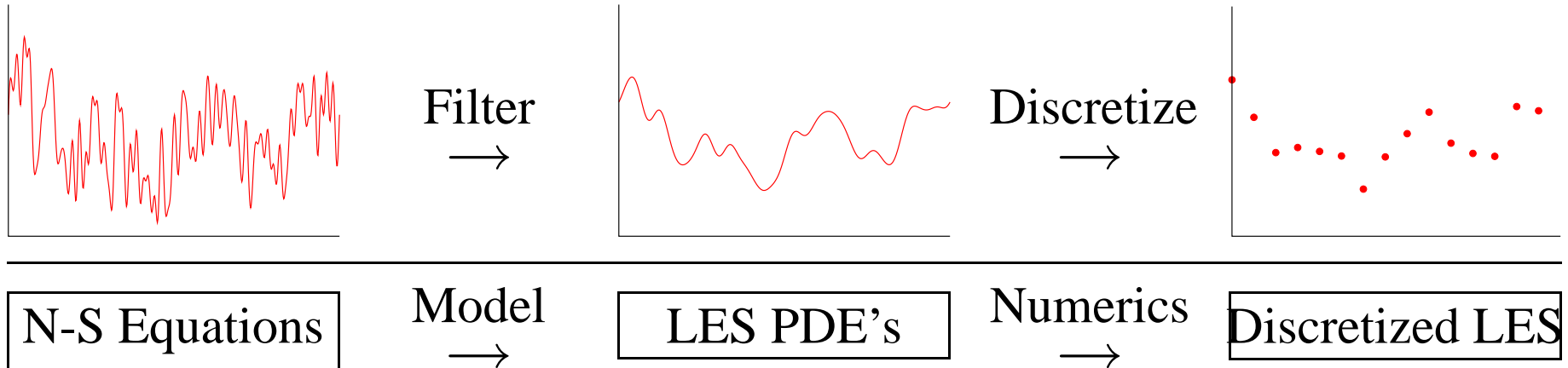
Optimal LES Development Map



Filtering and LES

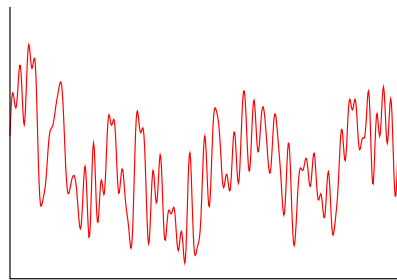
- Filters precisely define the large scales to be simulated
 - Not absolutely necessary, but useful
 - Provides a framework in which to develop models
- Two flavors of filtering and LES
 - Continuously filtered LES
 - Discretely filtered LES

Continuous LES

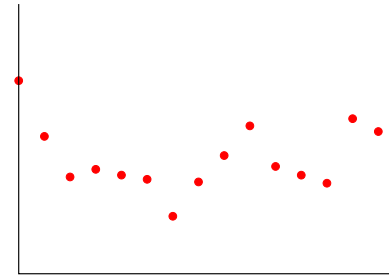


- Many filters are invertible or nearly so (e.g. Gaussian)
- A hypothetical exercise: suppose filter can be inverted
 - Determine evolution by defiltering and advancing N-S
 - Best “model” would be a DNS \Rightarrow DNS resolution
 - Coarse resolution determines accuracy limits
 - Best models must depend explicitly on discretization

Discrete LES



Filter
→



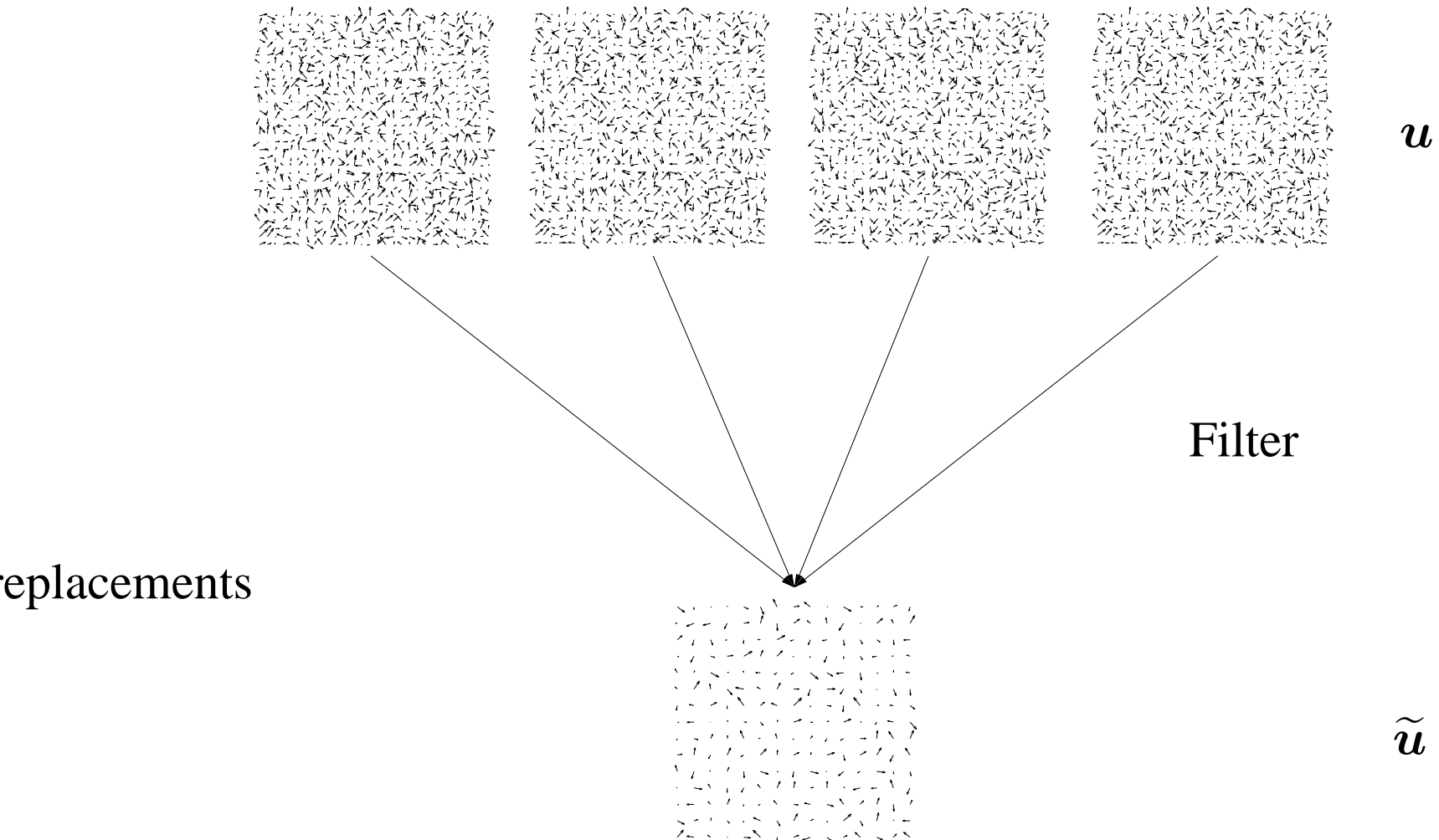
N-S Equations

Model
→

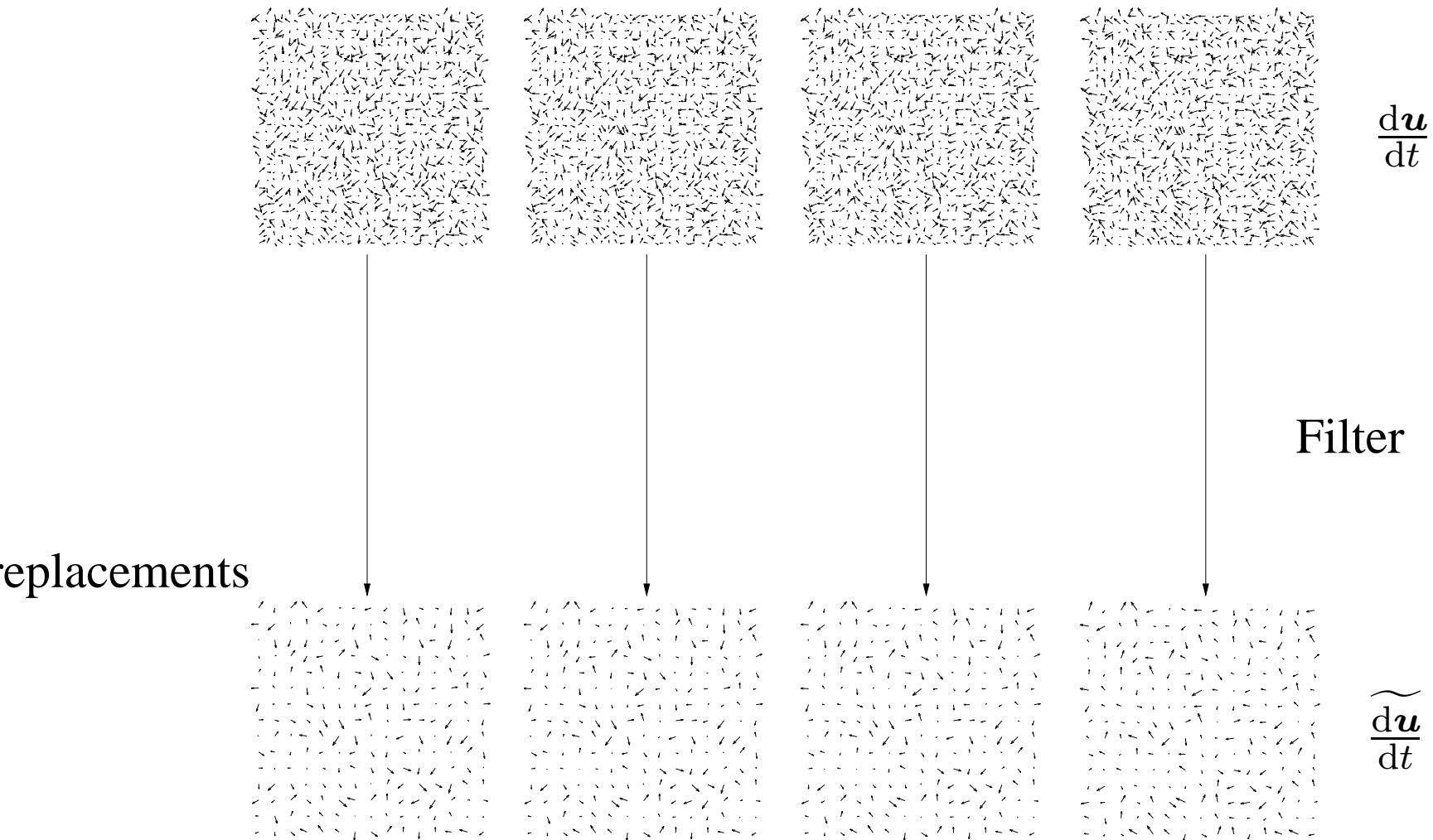
Discrete LES

- Examples: Fourier cut-off, sampled top-hat (finite volume), MILES
- Filter is not invertible & stochastic modeling tools are applicable
 - Many turbulent fields map to same filtered state
 - Evolution of filtered state considered stochastic
- This is the formulation used here

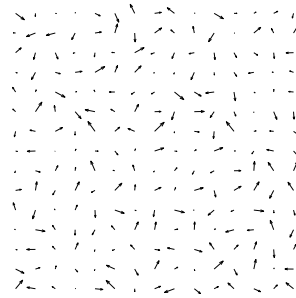
Stochastic Evolution of LES



Stochastic Evolution of LES

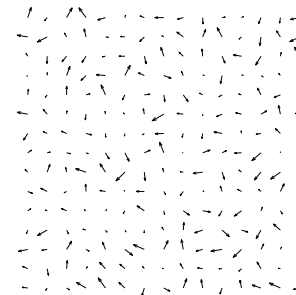
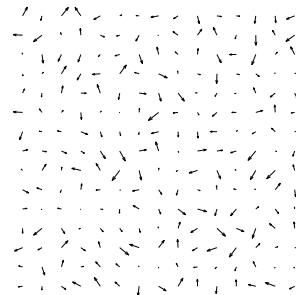
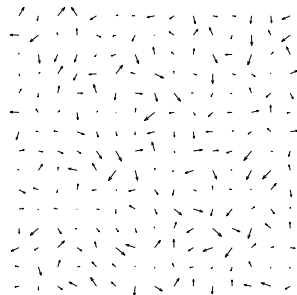
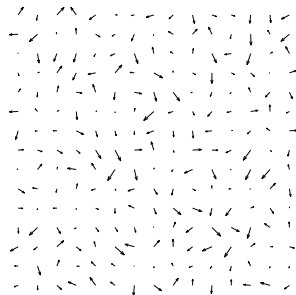


Stochastic Evolution of LES


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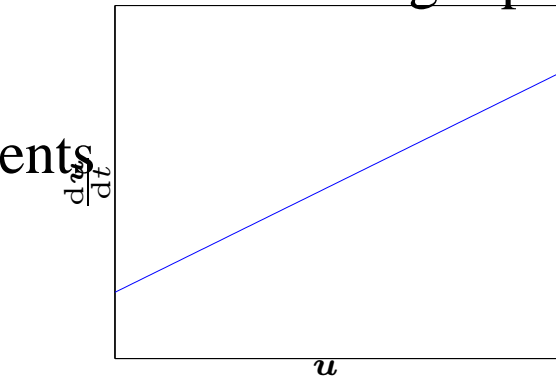
Mapping from filtered field to filtered evolution?

replacements

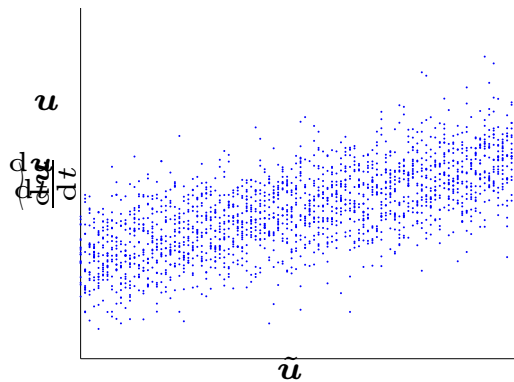

 $\widetilde{\frac{du}{dt}}$

Ideal LES

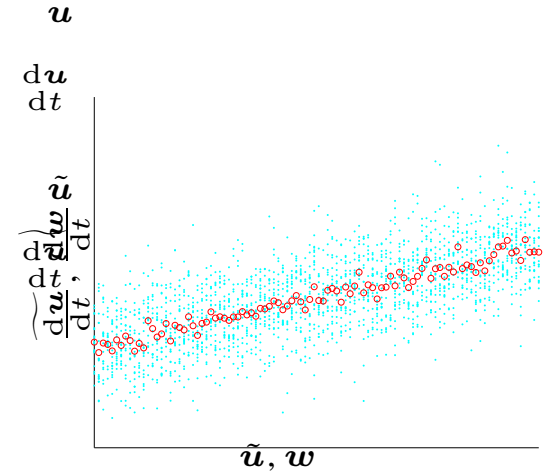
PSfrag replacements



Turbulence



Filtered Turbulence



Ideal LES

- Best deterministic LES evolution: Average of filtered evolutions of fields mapping to the current LES state

$$\frac{d\mathbf{w}}{dt} = \left\langle \frac{d\tilde{\mathbf{u}}}{dt} \middle| \tilde{\mathbf{u}} = \mathbf{w} \right\rangle$$

- Equivalently average of model terms: $m = \langle M | \tilde{\mathbf{u}} = \mathbf{w} \rangle$
- Two Theorems:
 - 1) 1-time statistics of \mathbf{w} and $\tilde{\mathbf{u}}$ match (Pope 2000, Langford & Moser 1999)
 - 2) Mean-square difference between $\frac{d\mathbf{w}}{dt}$ and $\frac{d\tilde{\mathbf{u}}}{dt}$ minimized but finite.

Optimal LES

- Statistical data requirements for Ideal LES are outrageous
 - # of conditions = # DOF in LES
- Stochastic estimation as an approximation to conditional average
 - Pick functional form of $m(\boldsymbol{w})$
 - Minimize mean-square error of approximation to conditional average
 - Results in model formulation first proposed by Adrian (1979,1990)

Optimal LES

An example

- Estimate conditional average $m \approx \langle M | \tilde{u} = w \rangle$
- Suppose $m(w) = A + Bw + Cw^2 + Dw^3$, then

$$\langle (M - m(\tilde{u}))E_j \rangle = 0 \quad \Rightarrow \quad \langle ME_j \rangle = \langle m(\tilde{u})E_j \rangle$$

where $E = (1, \tilde{u}, \tilde{u}^2, \tilde{u}^3)$ is the event vector

- Equations solved for coefficients A, B, C and $D \Rightarrow$ Optimal model
- Must know $\langle ME_j \rangle$ and $\langle E_i E_j \rangle$
 - Try using DNS correlation data
 - Then get correlations from theory

Ideal vs. Optimal LES

- For a given turbulent flow and filter, Ideal LES is uniquely defined but unknown
- In contrast, several choices must be made to define Optimal LES
 - Selection of modeled term M
E.g. $\frac{d\tilde{u}}{dt}$, τ_{ij} or $\partial_j \tau_{ij}$
Matters because error minimized is different
 - Selection of model dependencies
E.g. spatial locality, nonlinearity
Matters because changes space in which minimum error is sought

Developing Optimal LES Models

■ Modeler needs to design the Optimal model

- Guidance provided by $\langle M E_j \rangle = \langle m E_j \rangle$
- Arrange so $\langle M E_j \rangle$ includes terms of dynamical interest

Model reproduces them *a priori*

Example: Terms in 2-point correlation or Reynolds stress equation

■ Statistical information required as input

- For quadratic estimates need correlations:

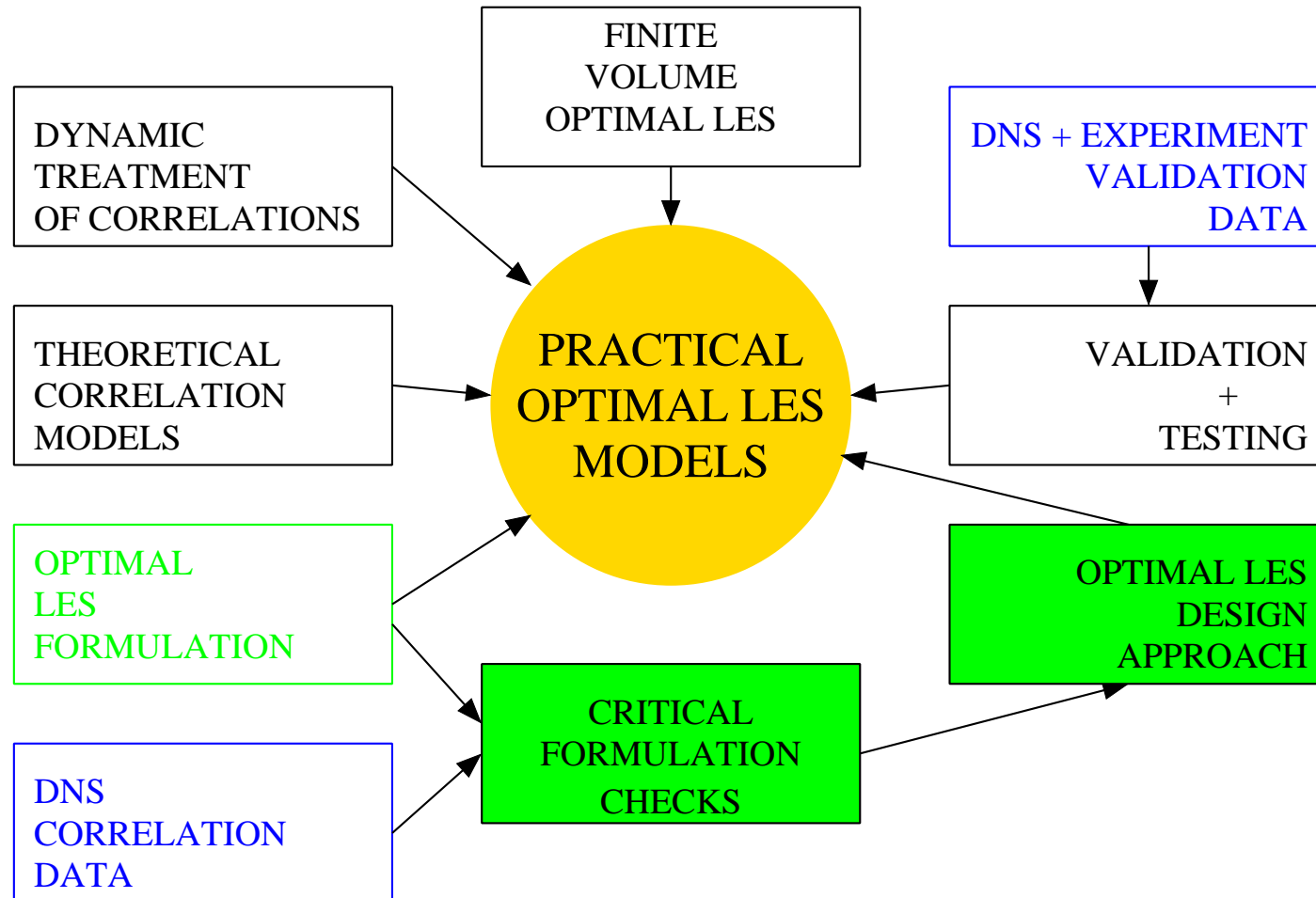
$$\langle u_i(\mathbf{x})u_j(\mathbf{x}') \rangle \quad \langle u_i(\mathbf{x})u_j(\mathbf{x}')u_k(\mathbf{x}') \rangle \quad \langle u_i(\mathbf{x})u_j(\mathbf{x}')u_k(\mathbf{x}'')u_l(\mathbf{x}'') \rangle$$

with separations of order the non-locality of the model

■ Use DNS correlations for testing,

■ Theoretically determined correlations later.

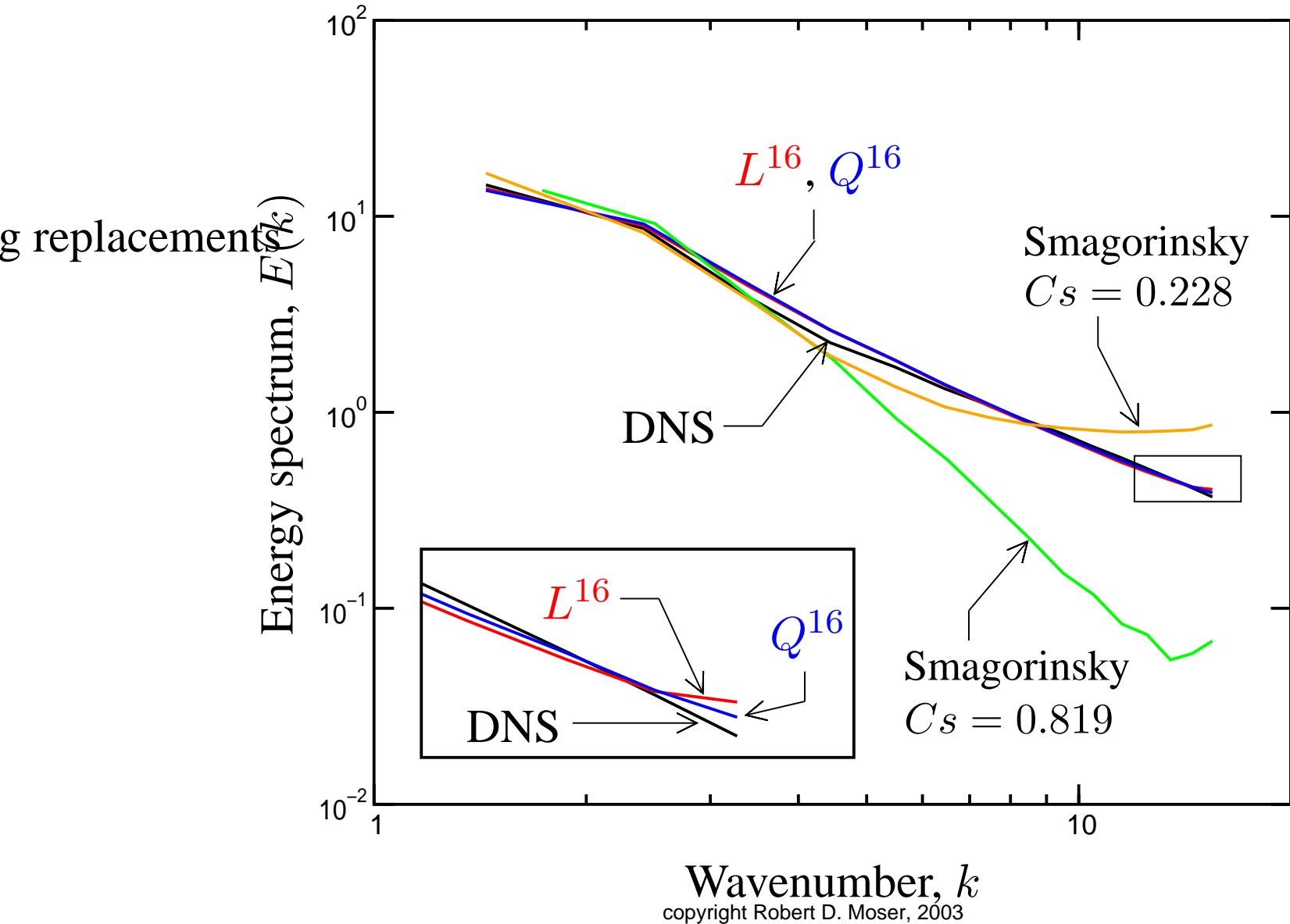
Optimal LES Development Map



Tests of Optimal LES with DNS Statistical Data

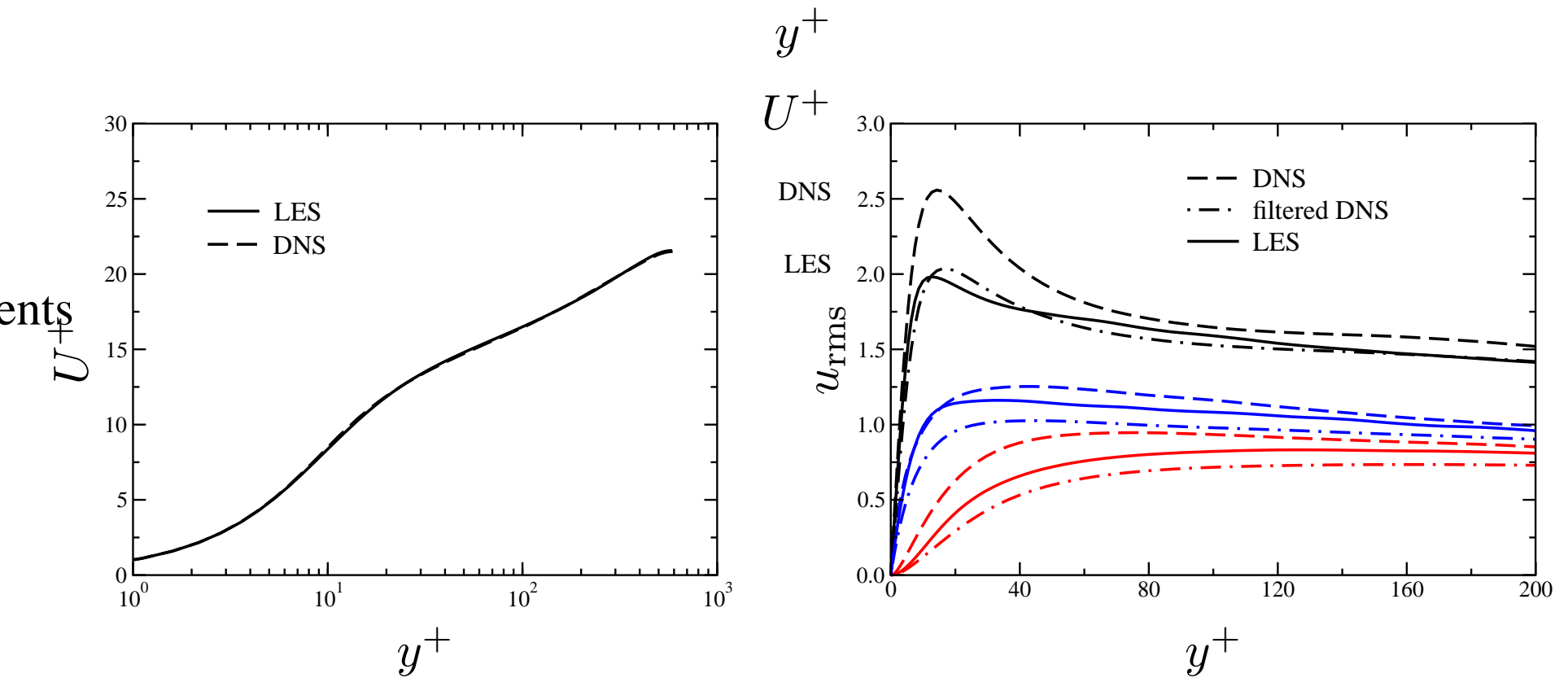
- Evaluate modeling approach without other uncertainties
- Principles of Optimal model design
- Test Cases:
 - Forced isotropic turbulence ($Re_\lambda = 164$)
Fourier cutoff filter
 - Turbulent flow in a plane channel ($Re_\tau = 590$)
Spectral representation/filter
Severely filtered ($\Delta x^+ = 116, \Delta z^+ = 58$)
 - Forced isotropic turbulence
Finite volume filter

Optimal LES of Forced Isotropic turbulence



Optimal LES of Turbulent Channel at $Re_\tau = 590$

PSfrag replacements



Constructing Good Optimal Models

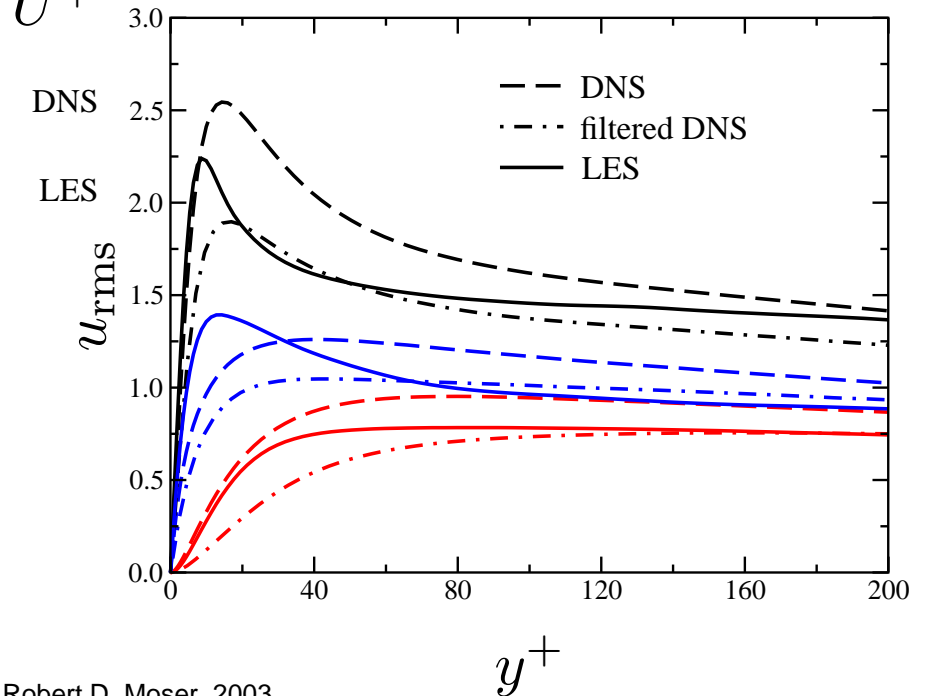
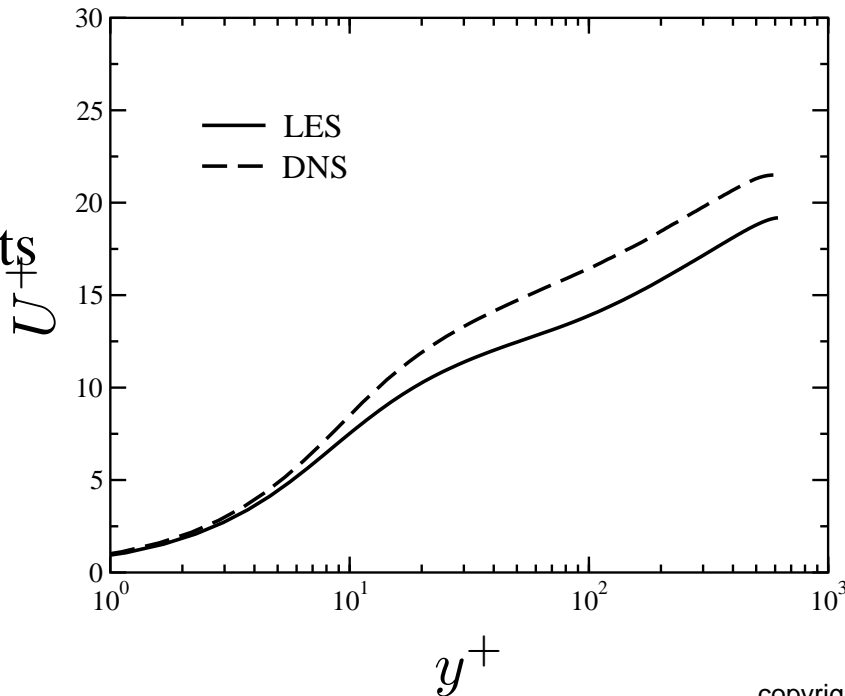
- Optimal model was formulated to reproduce the y -transport term in the Reynolds stress equation ($\partial_y u_k \tau_{i2}$)

PSfrag replacements

- Simpler optimal model that doesn't reproduce transport yields:

y^+

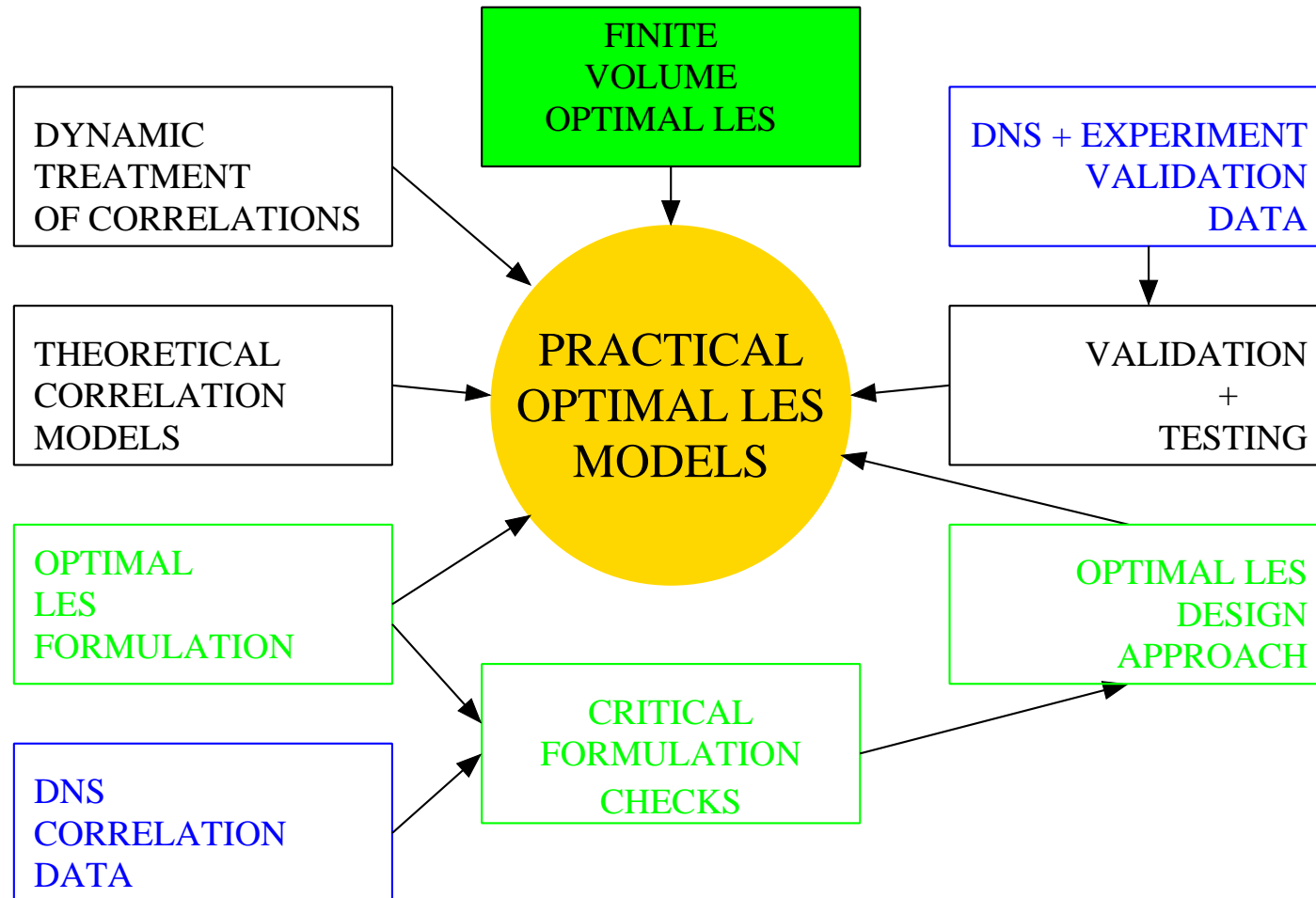
U^+



Responsibilities of an LES Model

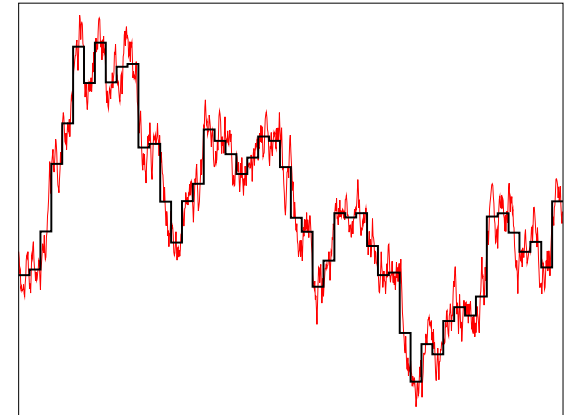
- An LES model must represent several effects of the subgrid turbulence
 - Dissipation of energy (and R_{ij}) - standard requirement
 - Subgrid contribution to mean equation (unresolved Reynolds stress).
 - Subgrid contribution to R_{ij} transport
 - Subgrid contribution to pressure redistribution of R_{ij}
- Optimal LES provides a mechanism to construct models that do this
 - Select M and E_j , so that $\langle M E_j \rangle$ includes terms in R_{ij} equation

Optimal LES Development Map



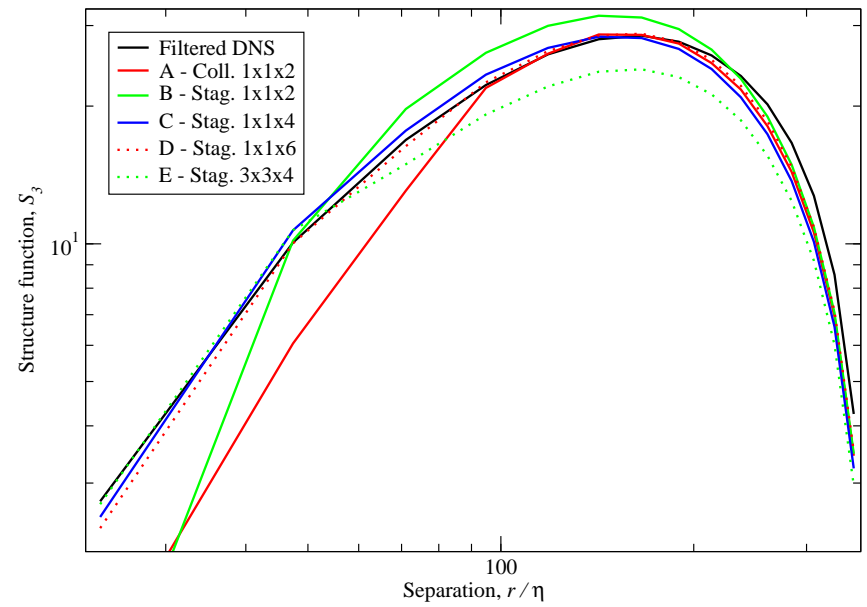
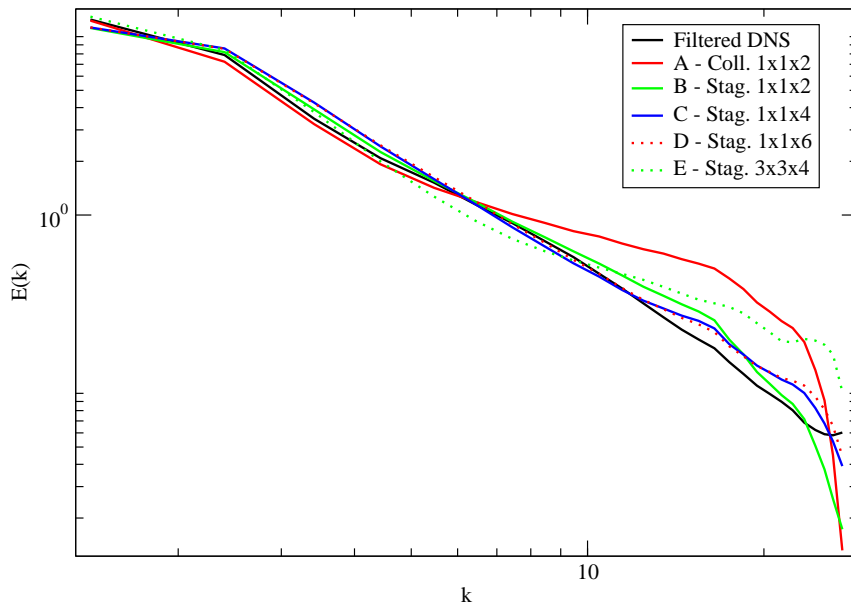
Finite Volume Optimal LES

- Like standard finite volume schemes, except:
 - Cell size not small compared to turbulence scales
 - Standard reconstruction techniques to determine finite volume fluxes are not applicable
 - True solution is not smooth on scale of the grid volume.
- Fluxes must be modeled.
 - Use Optimal model of the fluxes.
 - Estimate consistent with turbulence statistics, not numerical convergence.
- Need FV formulation for complex geometries
- Similar approach for Finite Difference and Finite Element discretizations

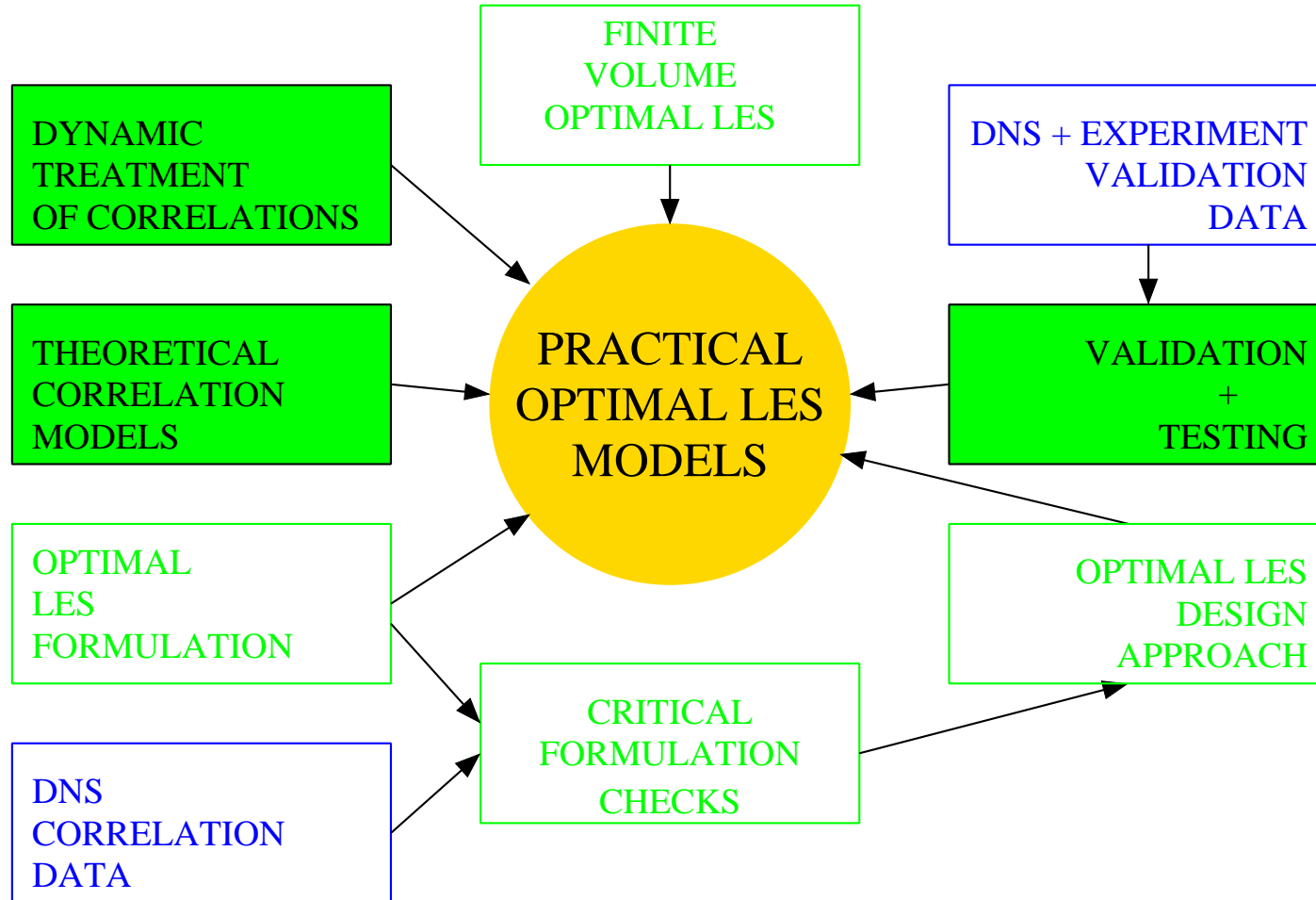


Performance of FV LES, $Re_\lambda = 164$

32^3 Isotropic LES



Optimal LES Development Map



Making Optimal LES Useful

- Simulations shown so far relied on DNS statistical data
 - Allowed properties and accuracy of OLES models to be explored
 - Allowed formulation details to be determined
 - Has not produced useful models
 - Need to do a DNS first
- Statistical input is needed
 - Rely as much as possible on theory

High Reynolds Number Optimal FV LES

- Estimation equations are of the form:

$$M'_{ij} = \sum_{\alpha} L'_{ijk} w_k^{\alpha} + \sum_{\alpha, \beta} Q^{\alpha\beta}_{ijkl} (w_k^{\alpha} w_l^{\beta})'$$

$$\langle w_m^{\gamma} M'_{ij} \rangle = \sum_{\alpha} L'_{ijk} \langle w_k^{\alpha} w_m^{\gamma} \rangle + \sum_{\alpha, \beta} Q^{\alpha\beta}_{ijkl} \langle (w_k^{\alpha} w_l^{\beta})' w_m^{\gamma} \rangle$$

$$\begin{aligned} \langle (w_m^{\gamma} w_n^{\delta})' M'_{ij} \rangle &= \sum_{\alpha} L'_{ijk} \langle w_k^{\alpha} (w_m^{\gamma} w_n^{\delta})' \rangle \\ &\quad + \sum_{\alpha, \beta} Q^{\alpha\beta}_{ijkl} \langle (w_k^{\alpha} w_l^{\beta})' (w_m^{\gamma} w_n^{\delta})' \rangle \end{aligned}$$

- **Green** terms are correlations of LES variables
Can compute from LES “on the fly” (dynamically)
- **Red** terms require modeling input

Modeling the Red Terms

- The **red** correlations are surface/volume integrals of:

$$\langle u_i(\mathbf{x})u_j(\mathbf{x})u_m(\mathbf{x}') \rangle \quad \langle u_i(\mathbf{x})u_j(\mathbf{x})u_m(\mathbf{x}')u_n(\mathbf{x}'') \rangle$$

- Assume $Re \rightarrow \infty$, separations in inertial range ($\mathbf{r} = \mathbf{x} - \mathbf{x}'$)
- Small-scale isotropy, Kolmogorov $\frac{2}{3}$ and $\frac{4}{5}$ laws, Quasi-normal approximation

$$\langle u_i(\mathbf{x})u_j(\mathbf{x}') \rangle = u^2 \delta_{ij} + \frac{C_1}{6} \epsilon^{2/3} r^{-4/3} (r_i r_j - 4r^2 \delta_{ij})$$

$$\langle u_i(\mathbf{x})u_j(\mathbf{x})u_m(\mathbf{x}') \rangle = \frac{\epsilon}{15} \left(\delta_{ij} r_m - \frac{3}{2} (\delta_{jm} r_i + \delta_{im} r_j) \right)$$

$$\begin{aligned} \langle u_i(\mathbf{x})u_j(\mathbf{x})u_m(\mathbf{x}')u_n(\mathbf{x}'') \rangle &= \langle u_i(\mathbf{x})u_j(\mathbf{x}) \rangle \langle u_k(\mathbf{x}')u_l(\mathbf{x}'') \rangle \\ &+ \langle u_i(\mathbf{x})u_k(\mathbf{x}') \rangle \langle u_j(\mathbf{x})u_l(\mathbf{x}'') \rangle \\ &+ \langle u_i(\mathbf{x})u_l(\mathbf{x}'') \rangle \langle u_k(\mathbf{x}')u_j(\mathbf{x}) \rangle \end{aligned}$$

Theoretical Optimal LES

■ Forced Isotropic Turbulence

- DNS at $Re_\lambda = 164$
- LES at $Re_\lambda = \infty$

PSfrag replacements

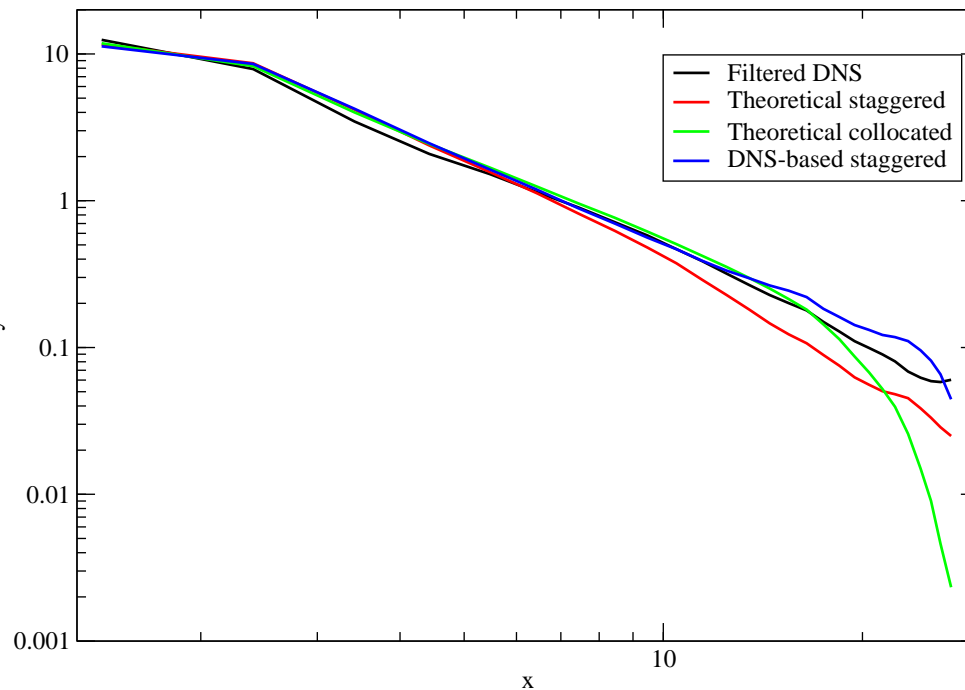
filtered DNS, $Re_\lambda = 164$

DNS-based optimal LES, $Re_\lambda = 164$

Theoretical optimal LES, $Re_\lambda = \infty$

Theoretical optimal LES, $Re_\lambda = 164$

k
 $E(k)$



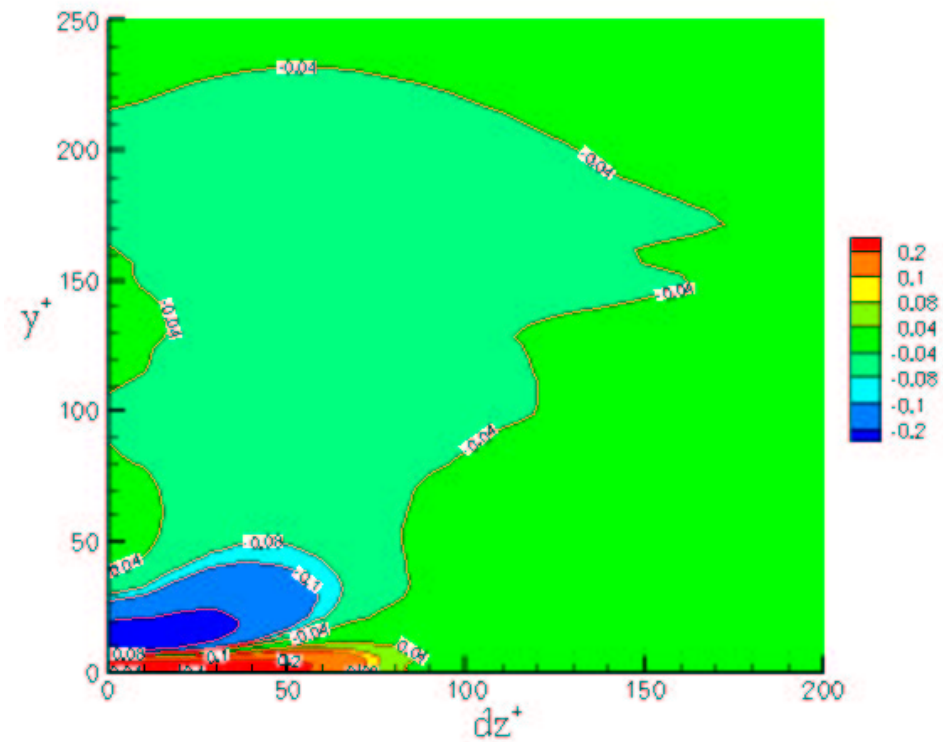
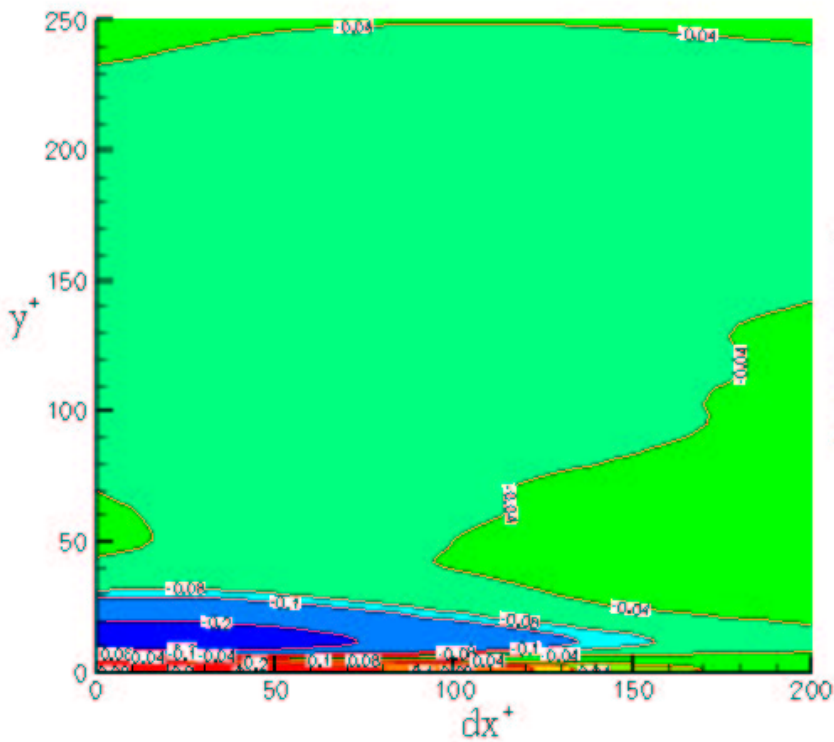
High (∞) Re Wall-Bounded Turbulence

- Assumptions of isotropy and inertial range not valid near wall
- **Green** terms can still be determined dynamically
- Need **red** correlations:
- A variety of modeling tools are being evaluated:
 - Log-layer similarity (Oberlack)
 - Anisotropy expansion & scaling (Procaccia)
 - Constraints from N-S equations
 - Quasi-Normal approximation

Test of Quasi-Normal Approximation

Channel Flow at $Re_\tau = 590$

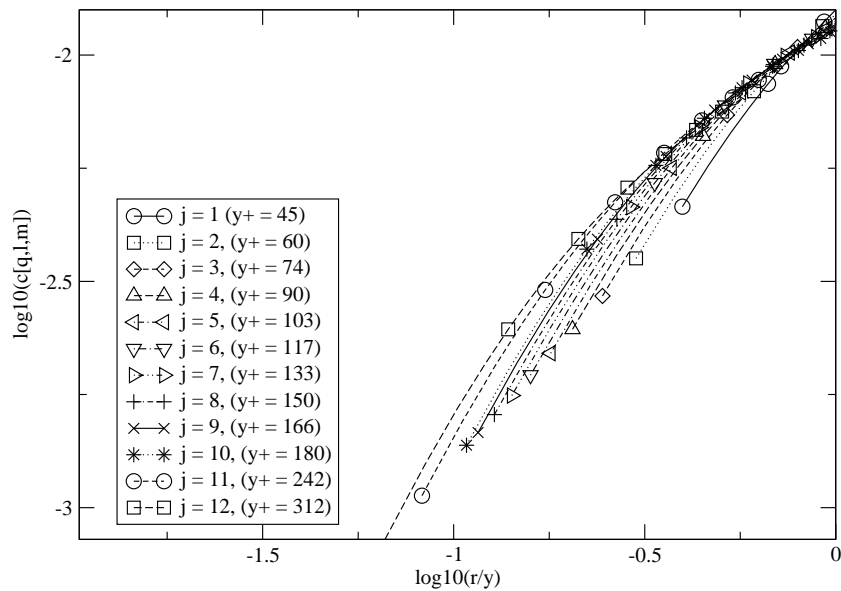
- Normalized error, $\phi_{11,11}(\mathbf{r}) = \frac{Q_{11,11}(\mathbf{r}) - Q_{NA}}{L(\mathbf{r})}$ where
 $L(\mathbf{r}) = \langle Q_{pq,rs}(\mathbf{r}) Q_{pq,rs}(\mathbf{r}) \rangle^{1/2}$



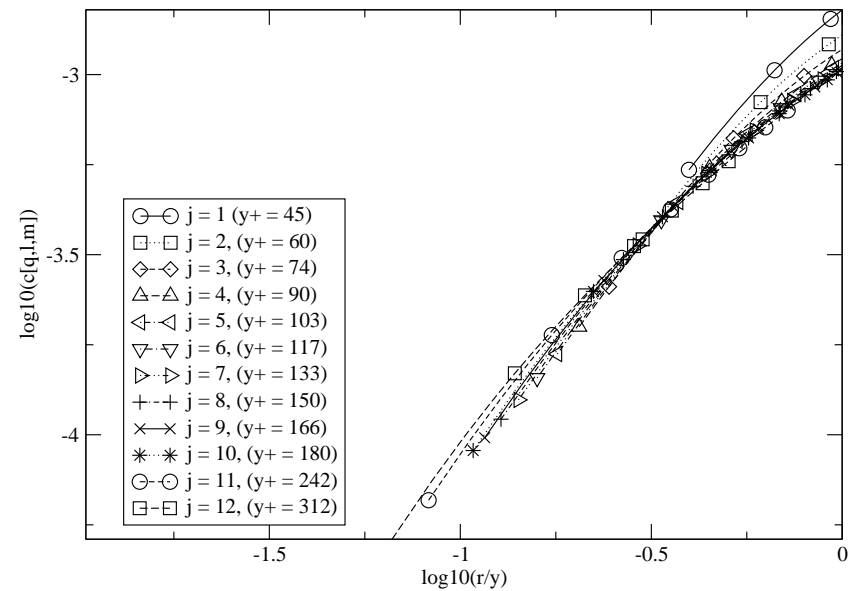
Similarity Scaling of Expansion Coefficients

in Channel at $Re_\tau = 940$, Expansion of Procaccia

$l = 0 \quad m = 0 \quad q = 1$



$l = 2 \quad m = 0 \quad q = 4$



Conclusions

- Discrete LES formulations are useful, avoid problems with discretization
- Optimal LES is a rational basis for discrete LES modeling
 - Yields remarkably good LES
 - But needs extensive statistical data as input
- For $Re \rightarrow \infty$, correlations available theoretically (away from walls)
 - Kolmogorov theory, Quasi-normal approximation, small-scale isotropy & a dynamic procedure.
- Near walls, need more information
- Also need models for subgrid contribution to statistical quantities of interest (e.g. turbulent energy).